IEEE

NATIONAL UNIVERSITY LVIV POLYTECHNIC 1816

IEEE UKRAINE SECTION

MEMS TECH

AGH

2021 IEEE XVIIth International Conference on the Perspective Technologies and Methods in MEMS Design (MEMSTECH)

Proceedings

Міністерство освіти і науки України
Національний університет «Львівська політехніка»
Західноукраїнський об'єднаний осередок IEEE

Матеріали
XVII-ої Міжнародної науково-технічної конференції

Перспективні технології і методи проектування МЕМС (MEMSTECH)

12-16 травня, 2021
Поляна, Україна

Львів, 2021

Investigation and Animation of Spring Dynamic Absorber

Yaroslav Pelekh
Department of Computational Mathematics and Programming
Lviv Polytechnic National University
Lviv, Ukraine
yaroslav.m.pelekh@lpnu.ua

Mykola Ihnatyshyn
Department of Mechanical Engineering, Natural Sciences and Information Technologies
Mukachevo State University,
Mukachevo, Ukraine
ignatishin.mykola@gmail.com

Roksolyana Stolyarchuk
Department of Computational Mathematics and Programming
Lviv Polytechnic National University
Lviv, Ukraine
roksoliana.r.stoliarchuk@lpnu.ua

Andrii Kunynets
Department of Computational Mathematics and Programming
Lviv Polytechnic National University,
Lviv, Ukraine
andrii.v.kunynets@lpnu.ua

Tania Magerovska
Department of information and analytical support of law enforcement agencies
Lviv State University of Internal Affairs
Lviv, Ukraine
tetiana.v.magerovska@lpnu.ua

Serhii Mentynskyi
Department of Computational Mathematics and Programming
Lviv Polytechnic National University
Lviv, Ukraine
sergii.m.mentynskyi@lpnu.ua

***Abstract*—Animations of complex mechanical systems with repetitive movements of interest in terms of activation of imagination in their study and research. Action of the spring dynamic absorber, investigation of the physical quantities of proposed mathematical model was implementated using non-linear Runge-Kutta type methods. Two kinds of methods: a nested method of relative accuracy 3(2) and two-sided method with accuracy 2 are consruted in proposed paper. Also estimation of main term of local error are obtained. For all these algorithms, in contrast to other algorithms, only three calls to the right-hand side of the differential equation are required.**

***Keywords*—spring dynamic vibration absorber, animation, Cauchy problem, nonlinear approximation, continued fractions, two-sided methods.**

I. Introduction

For investigation of applied mechanics problems it is useful to use modern mathematical modeling of vibrating technological systems [1]. The main problems of the dynamics for nonlinear oscillatory systems are the ability to create accurate analytical methods for studying such systems [2] only for a very limited class of mathematical models. Therefore, it is still urgent to create and develop new computational methods that allow to study, analyze and optimize the parameters of a wide range of vibrating technological systems. An important area of research in this topic is the synthesis and optimization of the parameters for mechanical vibration dampers, which are now widely used [3]. They are designed for dampen vibrations of mechanisms, machines [4], building structures. Also, when studying the parameters of dynamic systems and calculating the strength of the structure, it is important to take into account the influence of adverse environmental factors [5, 6].

In the study of known designs of mechanical oscillation valves, it is necessary to provide the relationship that combines the dynamics of linear and nonlinear oscillations with their structural and mechanical characteristics. On the basis of the received mathematical formulas it is possible to set new optimization problems.

The purpose of the study is to construct a mathematical model of a passive elastic dynamic dampener and to investigate the behavior of dynamic systems using two-sided methods. The investigated mechanical vibration dampers belong to two-mass mechanical systems. Such systems are being actively researched with the aim to optimize them. In [7], a parameter extension method for vibration impact systems while modeling the impact of contact force, was developed, theoretical calculations were designed for two-mass systems with two degrees of freedom. In [8] studies were conducted to reduce the noise of internal combustion engines by means of mechanical damping devices and the design of one of the possible variants of such device was developed. In [9, 10] the pendulum dumbbell type damper is investigated and it is proposed to adjust the natural frequency of the pendulum dampener with the help of optimal choice of design dimensions. In [11-15], investigations of cylindrical, dynamic mechanical vibration dampers, methods of protection against vibrations and shocks were carried out.

Since fractional-rational methods along with two-sided methods give good convergence of algorithms, they were used for research in the proposed work [16-18].

II. Formulation of the Problem

The motion of a two-mass system of a passive spring dynamic vibration absorber [7] (Fig. 1) is described by a system of two second-order differential equations, which can be we reduce to four first-order differential equations with zero initial values:

$$\frac{dx_1}{dt} = v_1, \qquad \frac{dx_2}{dt} = v_2,$$

$$\frac{dv_1}{dt} = Q(t)/m_1 - \left(\omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) + 2r_1 v_1\right),$$

$$\frac{dv_2}{dt} = \omega_2^2 (x_1 - x_2) - 2r_2 v_2. \qquad (1)$$

Now we need to find a solution to this system, as well as to investigate and analyze the physical quantities of the spring dynamic absorber and create an animation program for the dynamic system of a passive spring damper.

978-1-6654-2410-3/21/$31.00 ©2021 IEEE

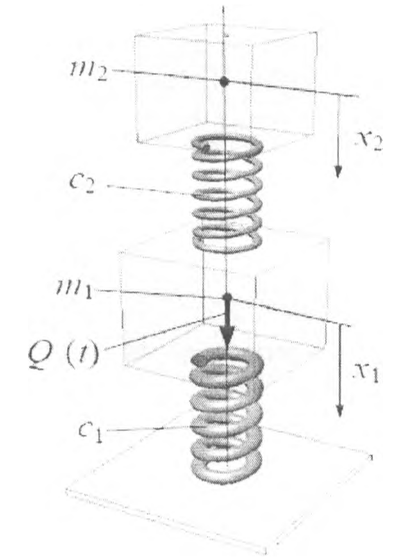

Fig. 1 The passive dynamic spring vibroabsorber

III. Construction of the Third-Order Accuracy Method

Consider problem (1) in a more general formulation, namely, as a Cauchy problem for system of differential equations:

$$\frac{d\bar{u}}{dt} = \bar{f}(t,\bar{u}), \quad \bar{u}(t_0) = \bar{u}_0, \quad t \in [t_0, t_0 + T], \quad (2)$$

where

$$[t_0, t_0 + T] \subset R, \ \bar{u}(t) \in R^S,$$
$$f : [t_0, t_0 + T] \times R^S \to R,$$

and f has the necessary smoothness for the calculations.

Since the proposed method is component in nature, for simplicity, we can consider one differential equation.

According to the theory of construction of one-step Runge-Kutta methods [19], we will look for a numerical solution of the corresponding Cauchy problem in the form of expansion of the exact solution by means of continuous fractions [20]:

$$u_{n+1}^{[k,l]} = u_n \Big/ \psi_n^{[k,l]}, \quad (3)$$

where

$$\psi_n^{[k,l]} = \sum_{i=0}^{k-1} \psi_{i,0} + \cfrac{\psi_{k,0}}{1 + \cfrac{\psi_{k,1}}{1 + \ddots + \cfrac{\psi_{k,l-1}}{1 + \psi_{k,l}}}}. \quad (4)$$

If $k + l = 3$ ($k = 1,2,3$; $l = 0,1,2$), then the expressions for $\psi_{k,l}$ are of the form:

$$\psi_{i,0} = -\frac{\sigma_m}{u_n} \sum_{m=1}^{i} \psi_{i-m,0}, \quad i = \overline{1,3}; \quad u_n \neq 0,$$

$$\psi_{2,1} = -\frac{\psi_{3,0}}{\psi_{2,0}}, \quad \psi_{1,2} = \psi_{2,1} - \psi_{1,1},$$

$$\sigma_m = \tau \cdot \sum_{i=1}^{3} c_{mi} k_i, \ k_i = f(t_n + a_i \tau, u_n + \tau \sum_{j=1}^{i-1} b_{ij} k_j),$$

$$a_1 = 0, \quad a_i = \sum_{j=1}^{i-1} b_{ij}. \quad (5)$$

Here τ – integration step $(\tau = t_{n+1} - t_n, n = 0,1,2,\ldots)$, c_{mi} $(m,i = \overline{1,3})$, a_i, b_{ij} $(i = 2,3;\ j = \overline{1,i-1})$ parameters.

Theorem. If parameters c_{mi} $(m,i = \overline{1,3})$, a_i, b_{ij} $(i = 2,3;\ j = \overline{1,i-1})$ satisfy the system of algebraic equations

$$\sum_{i=1}^{3} c_{1i} = 1, \quad \sum_{j=1}^{3} c_{ij} = 0, \quad i = 2,3,$$

$$\frac{1}{2} - \sum_{i=2}^{3} c_{1i} a_i - \sum_{i=2}^{3} c_{2i} a_i = 0, \quad \sum_{i=2}^{3} c_{3i} a_i = 0,$$

$$\frac{1}{6} - \sum_{m=2}^{3} \sum_{i=2}^{3} c_{mi} \frac{a_i^2}{2} = 0, \quad a_i = \sum_{j=2}^{i-1} b_{ij}, \ (i = \overline{2,3}),$$

$$\frac{1}{6} - \sum_{m=2}^{3} \sum_{i=2}^{3} c_{mi} \sum_{j=2}^{i-1} b_{ij} a_j = 0, \quad (6)$$

then there is an assessment

$$R_{n+1}^{[k,l]} = u(x_{n+1}) - u_{n+1}^{[k,l]} = O(\tau^4). \quad (7)$$

This system has three families of solutions. Here is one of them:

1) If $a_2 \cdot a_3 (a_3 - a_2)(a_2 - 2/3) \neq 0$, then

$$c_{11} = 1 + \frac{2 - 3(a_2 + a_3)}{6 a_2 a_3} - c_{33} \frac{a_3 - a_2}{a_2} + c_{22} + c_{33},$$

$$c_{12} = \frac{3a_3 - 2}{6a_2(a_3 - \alpha_2)} + c_{33} \frac{a_3}{a_2} - c_{22},$$

$$c_{13} = \frac{2 - 3a_2}{6a_3(a_3 - a_2)} - c_{23} - c_{33}, \quad (8)$$

$$c_{21} = -c_{22} - c_{23}, \ c_{31} = c_{33} \frac{a_3 - a_2}{a_2},$$

$$c_{32} = -\frac{a_3}{a_2} c_{33}, \qquad b_{21} = a_2,$$

$$b_{32} = \frac{a_3(a_3 - a_2)}{a_2(2 - 3a_2)}, \quad b_{31} = a_3 - b_{32},$$

where $c_{22}, c_{23}, c_{33}, a_2, a_3$ parameters;

In particular, if put $c_{ij} = 0$ for $j > i$, we will obtain respectively:

1a) $$c_{11} = 1, \quad c_{21} = -\frac{1}{2a_2}, \quad c_{12} = c_{13} = 0,$$

$$c_{22} = \frac{1}{2\alpha_2}, \quad c_{23} = 0, \quad c_{31} = \frac{2 - 3a_2}{6a_2 a_3},$$

$$c_{32} = \frac{3a_2 - 2}{6a_2(a_3 - a_2)}, \quad c_{33} = \frac{2 - 3a_2}{6a_3(a_3 - a_2)} \quad (9)$$

$$b_{21}=a_2,\qquad b_{32}=\frac{a_3(a_3-a_2)}{a_2(2-3a_2)},\qquad b_{31}=a_3-b_{32}.$$

IV. Second Order of Accuracy Methods with Two-Sided Estimation of Local Error

We construct a two-sided method. Consider formulas (3)-(5) for $k=3$, $l=0$ and put

$$c_{11}=1,\quad c_{12}=c_{13}=c_{23}=0,\quad a_2=b_{21},\quad a_3=b_{31}+b_{32}.$$

Let us write three sets of parameters that give a two-sided estimation of the solution at each nodal point:

I. $$c_{21}=-\frac{1+\gamma^{[\pm]}}{2a_2},\qquad c_{22}=\frac{1+\gamma^{[\pm]}}{2a_2},$$

$$c_{31}=\frac{2+3\left(\gamma^{[\pm]}a_3-a_2\right)}{6a_2a_3},\qquad c_{32}=\frac{3a_2-2-3\gamma^{[\pm]}(a_3-a_2)}{6a_2(a_3-a_2)},$$

$$c_{33}=\frac{2-3\alpha_2}{6\alpha_3(\alpha_3-\alpha_2)},\quad b_{21}=a_2,$$

$$b_{32}=\frac{a_3}{a_2}\cdot\frac{a_3-a_2}{2-3a_2},\qquad b_{31}=a_3-b_{32},\qquad (10)$$

moreover $a_2a_3(a_3-a_2)(2-3a_2)\neq 0$.

Local error of these formulas looks like :

$$R_{n+1}^{[3,0]}=\gamma^{[+]}\tau^2k_1(k_2-k_1)/(a_2u_{n-1})+O\left(\tau^4\right).\qquad (11)$$

These formulas that correspond to two values of $\gamma^{[+]}$ are two-sided formulas and differ only in sign, one of which gives the upper approximation and the other the lower approximation. As an approximate solution, the half-sum of these two-sided approximations is taken.

The proposed algorithm allows us to find several approximations to the exact solution, comparing which we can get usefull information for selection of the integration step.

V. Investigation Results

The calculations were carried out in the Mathcad package. Initial data (Fig.1) :

$m_1=100\,kg$, $m_2=10\,kg$ – masses of the system of bodies,

$c_1=10\,N/m$, $c_2=1\,N/m$ – stiffness of springs,

$\beta_1=1.5\cdot10^{-1}\,kg/s$, $\beta_2=1.5\cdot10^{-1}\,kg/s$ – oscillation damping parameters,

$H=10N$ – amplitude of disturbing force,

$f=0.05\,Hz$ – frequency of disturbing force.

Formulas for calculation:

$$\omega_1=\sqrt{c_1/m_1},\qquad \omega_2=\sqrt{c_2/m_2},$$
$$r_1=\beta_1/(2m_1),\qquad r_2=\beta_2/(2m_2),$$
$$h=H/m_1,\qquad \omega=2\pi f.\qquad (12)$$

The disturbing force is related to the mass, $f(t)=Q(t)/m_1=h\cdot\sin(\omega t)$.

In our model it is assumed that the resistance of the medium is negligible and has little effect on the resonant frequencies of the dampers and bodies of mass m_1, whose damping oscillations are being investigated.

Graphical representation of numerical solutions positions and velocities of bodies of mass m_1 and m_2 (Fig. 2-Fig. 5). From the graphs presented in Fig. 2 and Fig. 3, we see that the amplitude and velocity of the body with mass m_1 reduced.

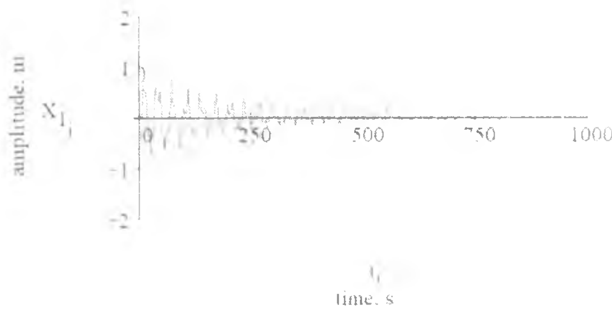

Fig. 2 Graph of the amplitude of oscillations of the body with mass m

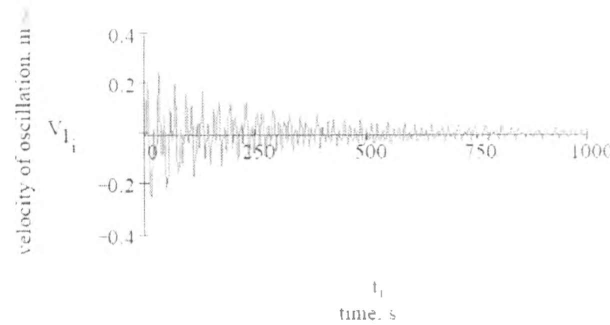

Fig. 3. Graph of the velocity of oscillations of the body mass m_2

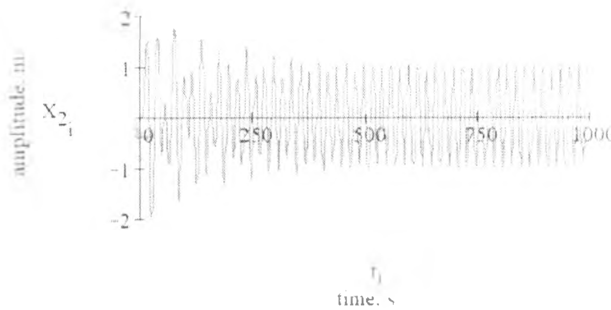

Fig. 4 Graph of amplitude of oscillations of the damper (body mass m2)

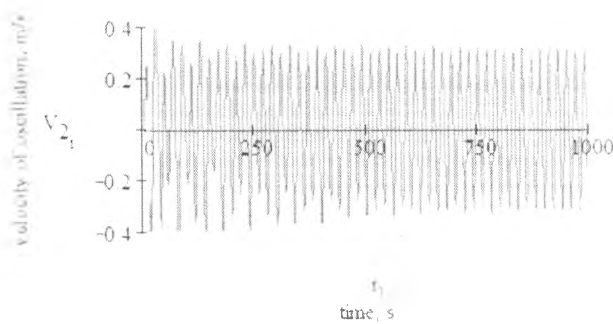

Fig. 5 Graph of the speed of oscillations of the damper

The amplitude of oscillations and velocities of the body mass m2 (vibration dampener) is stabilized at a certain level and shifts in phase relative to the oscillations of the body m_1 (Fig. 4, 5), which can be observed in the animation and there is a redistribution of energy between the body and the oscillator dampener – energy goes to the mass dampener m_2 (Fig. 6).

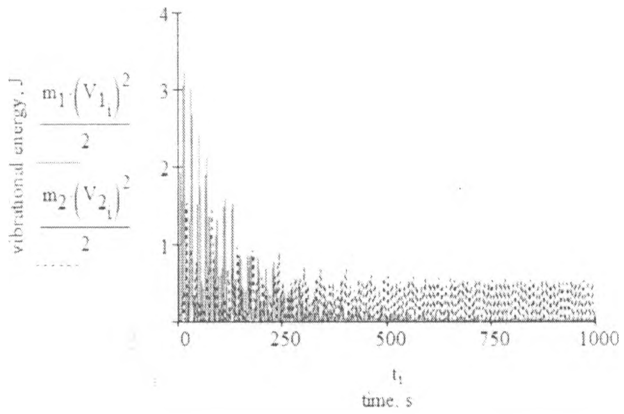

Fig. 6. Fluctuation energy transition from the body of mass m1 to the body mass m2

Initially, both bodies oscillate by the phase shift ≈0, and at the moment of maximum damping of oscillations, the oscillation dampener moves in opposite phase to the mass m_1.

Compiled a program for animation of dynamic system a passive spring absorber [21].

VI. Conclusion

Action of a spring dynamic absorber using the proposed methods, as well as the constructed model were investigated on animation in Mathcad.

The third-order accuracy method is constructed, as well as the second-order accuracy formulas, which at each integration point give an approximation to the exact solution with excess and deficiency.

Without additional calculation of the right-hand side of the differential equation, we obtain the estimation of the local error principal term.

Derived two-sided formulas allow at each step of integration to reduce the number of appeals to the right hand-side of the differential equation by 25% compared to the traditional two-sided Runge-Kutta methods [22-25].

The obtained program makes it possible to simulate the operation of the spring dynamic absorber under different external influences, not only with harmonic periodic force, but also with shocks, vibrations with different nature and law of the resistance of the environment.

References

[1] P. Ya. Pukach and I. V. Kuzio, "Nonlinear transverse vibrations of semiinfinite cable with consideration paid to resistance", Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, No. 3, pp. 82-86, 2013, [in Ukrainian]

[2] Z. Nytrebych, O. Malanchuk, V. Il'kiv, and P. Pukach, "On the solvability of two-point in time problem for PDE", Italian J. of Pure and Appl. Math., Vol. 38, pp.715–726, 2017.

[3] Ya. Pelekh, I. Konyk, M. Hlobchak, G. Cherchyk, V. Opalko, and B. Diveeyv "Optimization of the Particle Vibration Absorbers as SDOF System". Perspective Technologies and Methods in MEMS Design, MEMSTECH 2017 - Proceedings of 13th International Conference (Polyana, April 20-23, 2017), pp. 157-160, 2017

[4] Z. Nytrebych, V. Ilkiv, P. Pukach, O. Malanchuk, I. Kohut, and A. Senyk, "Analytical method to study a mathematical model of wave processes under two-point time conditions", Eastern-European Journal of Enterprise Technologies, Vol. 1, Is. 7(97), pp. 74-83, 2019.

[5] Yu Prokopovych, Ya. Pelekh, I Kostenko, and I Zashkilnyak, "Mathematical model and boundary equilibrium of the cylindrical shell with elastic filling depended by the internal crack". Perspective Technologies and Methods in MEMS Design, MEMSTECH 2018 Proceedings of 14th International Conference (Polyana, April 18-22) Lviv, pp. 136-140, 2018

[6] I Kostenko, Ya Pelekh, T Nykolyshyn, and V Karkuliovskyy, "Modelling and method of investigation of the limiting equilibrium of an anisotropic cylindrical shell with crack", Perspective Technologies and Methods in MEMS Design, MEMSTECH 2017 - Proceedings of 13th International Conference (Polyana, April 20-23) Lviv, pp. 146-150, 2017

[7] V A Bazhenov, O S Pogorelova, and T G Postnikov, "Development of a parameter extension method for vibration impact systems in contact force interaction impact modeling". Opir materialiv i teoriya sporud, No 87, pp 62-72 , 2011, [in Ukraine]

[8] F M. Sudak, I. F. Voronin, and D P Altukhov, "Noise reduction of internal combustion engines by mechanical damping devices". Visti Avtomobilno-dorozhnoho instytutu, No 2(8), pp 98-101, 2009, [in Ukraine]

[9] V. P Legeza, and D. V Legeza, "Mathematical model of a dynamic system with a two-mass pendulum". Problemy prochnosti, No 6, pp 100-109, 2008, [in Russian]

[10] V P Legeza, and S V. Guzenko, "A mathematical model of the dynamic behavior of a vibration protection system and its pendulum dumbbell type dampener", Naukovi pratsi Natsionalnoho universytetu kharchovykh tekhnolohiy, No 16, pp 149-152, 2005, [in Ukraine]

[11] Vibration in technology, T 6. Protection against vibration and shock reference book / ed. K V Frolovna, Moskva, Mashinostroyeniye, 456 p., 1981, [in Russian]

[12] B. G Korneev, and L M. Reznikov, Dynamic vibration dampers, Moskva, Nauka, 304 p., 1988, [in Russian]

[13] A A Klimenko, and Yu V Michlin "Nonlinear vibration modes of a mechanical system with a pendulum vibration damper", Mekhanika tverdogo tela, Issue 40, pp 162-171, 2010, [in Russian]

[14] M I Ignatyshyn and P F. Kholod, "Modeling of the mechanical oscillator in the Mathcad system", Visnyk Natsionalnoho universytetu "Lvivs'ka politekhnika", Teoriya ta praktyka budivnytstva, No 742, pp. 94–98, 2012, [in Ukraine]

[15] I I. Sidorenko, R V. Komar, and O. A. Tkachov, "Spring dynamic vibration absorber with mechanical feedback", Bulletin of the CSTU Engineering Sciences, Kharkiv, Issue 139, pp 103-107, 2013, [in Ukraine]

[16] A I Aptekarev and M L Yattselev, "Pad e approximants for functions with branch points-strong asymptotics of Nuttall Stahl polynomials" Acta Math., 215, 217–280, 2015

[17] D Butusov, A Karimov, A Tutueva, D Kaplun, and E. Nepomuceno, "The Effects of Pade numerical integration in simulation of conservative chaotic systems", Entropy, 21(4) 362-369, 2019.

[18] Y M. Hlynskyi and Y. Y. Panchyshyn, "On the monotonicity and constancy of signs of some rational explicit methods for nonlinear system of ordinary differential equations", Ukrainian Mathematical Journal, 47, pp 976-981, 1995

[19] J C Butcher, Numerical Methods for Ordinary Differential Equations, London, Wiley & Sons, 2008

[20] Ya. M. Pelekh, "An approach to deducing approximate solution to the Cauchy problem for nonlinear differential equations", Ukrainian Mathematical Journal, Vol. 44, Issue 12, pp 1554-1560, 1992

[21] https://www.youtube.com/watch?v=-Vv41D-OAPc

[22] B S Dobronets and V. V Shaidurov, Bilateral numerical methods, Novosibirsk, Nauka, 1990, [in Russian]

[23] N S Bakhvalov, Numerical Methods, v. I, Moscow, Science, 1975 [in Russian]

[24] V. I. Krylov, V V Bobkov, and P. I Monastyrnyi, Computational Methods, Vol II, Moscow, Nauka, 1977 [in Russian]

[25] I I Lyashko, V I Makarov, and A. A Skorobogat'ko, Methods of Calculating, Kiev, High school, 1977 [in Russian]